Notez bien que ce n'est pas Maple qui fais les exercices, c'est la personne qui utilise Maple qui fait les exercices, mais en gagnant du temps sur des trucs de calculs, de vérifications, de factorisations, de simplifications...

```
> restart;# sol exo 1
```

(i)

```
> assume(a>0,b>0);
> Int(Int(1,y=-b*sqrt(1-x**2/(a**2))..b*sqrt(1-x**2/(a**2))),x=-a.
  .a)=int(int(1,y=-b*sqrt(1-x**2/(a**2))..b*sqrt(1-x**2/(a**2))),x
  =-a..a);
```

$$\int_{-a\sim}^{a\sim}\int_{-b\sim\sqrt{1-\frac{x^2}{a\sim^2}}}^{b\sim\sqrt{1-\frac{x^2}{a\sim^2}}} 1\,dy\,dx = a\sim\,\pi\,b\sim$$

(iii)

```
> restart;assume(a>0,b>a,c>0,d>c);
> u=y/(x**2);v=x*y;
```

$$u = \frac{y}{x^2}$$

$$v = x\,y$$

```
> x:=(u/v)**1/3;y:=(u*v**2)**(1/3);
```

$$x := \frac{u}{3\,v}$$

$$y := (u\,v^2)^{(1/3)}$$

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
> jacobian([x,y],[u,v]);
```

$$\begin{bmatrix} \frac{1}{3\,v} & -\frac{u}{3\,v^2} \\ \frac{1}{3}\frac{v^2}{(u\,v^2)^{(2/3)}} & \frac{2}{3}\frac{u\,v}{(u\,v^2)^{(2/3)}} \end{bmatrix}$$

```
> det(%);# valeur absolue inutule, c'est déjà positif
```

$$\frac{1}{3}\frac{u}{(u\,v^2)^{(2/3)}}$$

```
> simplify(%);
```

$$\frac{1}{3}\frac{u}{(u\,v^2)^{(2/3)}}$$

```
> Int(Int(%,u=a..b),v=c..d)=int(int(%,u=a..b),v=c..d);
```

$$\int_{c\sim}^{d\sim}\int_{a\sim}^{b\sim}\frac{1}{3}\frac{u}{(u\,v^2)^{(2/3)}}\,du\,dv = -\frac{3\,(-a\sim^{(4/3)}+b\sim^{(4/3)})\,(-d\sim^{(1/3)}+c\sim^{(1/3)})}{4\,c\sim^{(1/3)}\,d\sim^{(1/3)}}$$

```
> simplify(%);
```

$$\frac{1}{3}\int_{c\sim}^{d\sim}\int_{a\sim}^{b\sim}\frac{u}{(u\,v^2)^{(2/3)}}\,du\,dv = -\frac{3\,(a\sim^{(4/3)}-b\sim^{(4/3)})\,(d\sim^{(1/3)}-c\sim^{(1/3)})}{4\,c\sim^{(1/3)}\,d\sim^{(1/3)}}$$

```
> restart;# sol exo 2
```

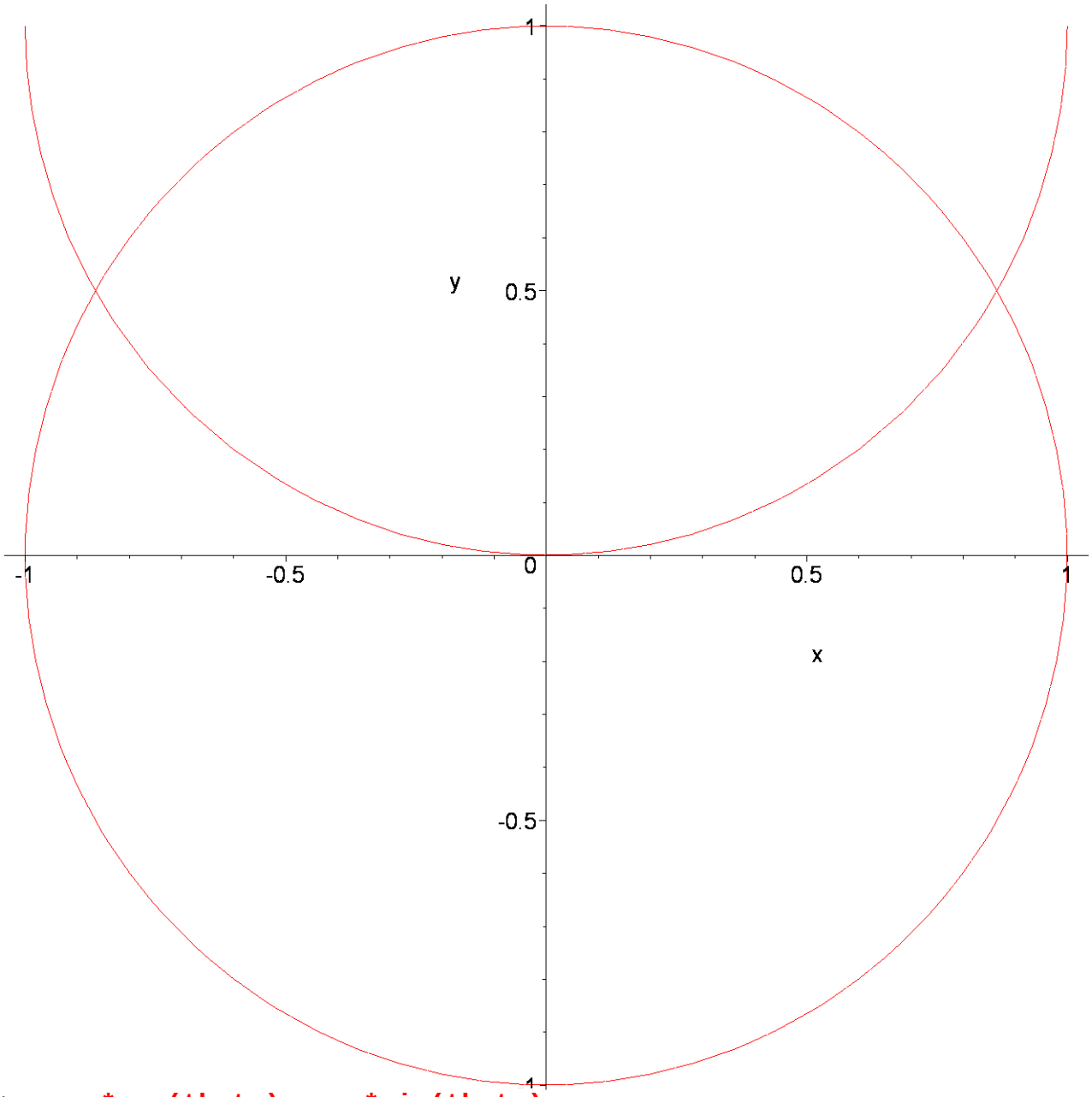
1)

```
> solve({x**2+y**2=1,x**2+y**2-2*y=0},{x,y});
```

$$\{y=\frac{1}{2},\ x=\frac{1}{2}\,\mathrm{RootOf}(_Z^2-3,\ label=_L1)\}$$

```
> with(plots):
Warning, the name changecoords has been redefined
```

```
> implicitplot([x**2+y**2=1,x**2+y**2-2*y=0],x=-1..1,y=-1..1);
```




```
> x:=r*cos(theta);y:=r*sin(theta);
```

$$x := r\cos(\theta)$$

$$y := r\sin(\theta)$$

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
```

```
> jacobian([x,y],[r,theta]);
```

$$\begin{bmatrix} \cos(\theta) & -r\sin(\theta) \\ \sin(\theta) & r\cos(\theta) \end{bmatrix}$$

```
> det(%);
```

$$\cos(\theta)^2 r + r\sin(\theta)^2$$

```
> simplify(%);
```

$$r$$

```
> x**2+y**2;
```

$$r^2 \cos(\theta)^2 + r^2 \sin(\theta)^2$$

```
> simplify(%);
```

$$r^2$$

```
> x**2+y**2-2*y;
```

$$r^2 \cos(\theta)^2 + r^2 \sin(\theta)^2 - 2\, r \sin(\theta)$$

```
> simplify(%);
```

$$r\,(-2\sin(\theta) + r)$$

```
> assume(-7*Pi/6<=theta,theta<=Pi/6);
> assume(max(0,2*sin(theta))<=r,r<=1);
> i:=Int(Int(r**2,r=max(0,2*sin(theta))..1),theta=-7*Pi/6..Pi/6)=i
  nt(int(r**2,r=max(0,2*sin(theta))..1),theta=-7*Pi/6..Pi/6);
```

$$i := \int_{-\frac{7\,\pi}{6}}^{\frac{\pi}{6}} \int_{\max(0,\,2\sin(\theta\sim))}^{1} r\sim^2\, dr\sim\, d\theta\sim = \int_{-\frac{7\,\pi}{6}}^{\frac{\pi}{6}} \frac{1}{3} - \frac{1}{3}\max(0,\,2\sin(\theta\sim))^3\, d\theta\sim$$

```
> i:=Int(Int(r**2,r=max(0,2*sin(theta))..1),theta=-7*Pi/6..-Pi)+In
  t(Int(r**2,r=max(0,2*sin(theta))..1),theta=-Pi..0)+Int(Int(r**2,
  r=max(0,2*sin(theta))..1),theta=0..Pi/6)=int(int(r**2,r=max(0,2*
  sin(theta))..1),theta=-7*Pi/6..-Pi)+int(int(r**2,r=max(0,2*sin(t
  heta))..1),theta=-Pi..0)+int(int(r**2,r=max(0,2*sin(theta))..1),
  theta=0..Pi/6);
```

$$i :=$$

$$\int_{-\frac{7\,\pi}{6}}^{-\pi} \int_{\max(0,\,2\sin(\theta\sim))}^{1} r\sim^2\, dr\sim\, d\theta\sim + \int_{-\pi}^{0} \int_{\max(0,\,2\sin(\theta\sim))}^{1} r\sim^2\, dr\sim\, d\theta\sim + \int_{0}^{\frac{\pi}{6}} \int_{\max(0,\,2\sin(\theta\sim))}^{1} r\sim^2\, dr\sim\, d\theta\sim$$

$$= 2\sqrt{3} + \frac{4\,\pi}{9} - \frac{32}{9}$$

```
> restart;# sol exo 3
```

1)

```
> x:=a*u*cos(v);y:=b*u*sin(v);
```

$$x := a\,u\cos(v)$$

$$y := b\,u\sin(v)$$

```
> assume(0<=u,u<=1);assume(0<=v,v<=2*Pi);
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected
> jacobian([x,y],[u,v]);
```

$$\begin{bmatrix} a\cos(v\sim) & -a\,u\sim\sin(v\sim) \\ b\sin(v\sim) & b\,u\sim\cos(v\sim) \end{bmatrix}$$

```
> det(%);
```

$$a\cos(v\sim)^2\,b\,u\sim + a\,u\sim\sin(v\sim)^2\,b$$

```
> simplify(%);
```

$$a\,b\,u\sim$$

```
> i:=Int(Int(a*b*u*sqrt(1-u**2),u=0..1),v=0..2*Pi)=int(int(a*b*u*s
  qrt(1-u**2),u=0..1),v=0..2*Pi);
```

$$i := \int_0^{2\pi}\int_0^{1} a\,b\,u\sim\sqrt{1-u\sim^2}\,du\sim\,dv\sim = \frac{2\,a\,b\,\pi}{3}$$

```
> restart;# sol exo 6
```

1)

```
> i:=Int(Int(Int(1/((1+x+y+z)**3),z=0..1-x-y),y=0..1-x),x=0..1)=in
  t(int(int(1/((1+x+y+z)**3),z=0..1-x-y),y=0..1-x),x=0..1);
Warning, unable to determine if -1-x-y is between 0 and 1-x-y; try to use
assumptions or set _EnvAllSolutions to true
```

$$i := \int_0^{1}\int_0^{1-x}\int_0^{1-x-y} \frac{1}{(1+x+y+z)^3}\,dz\,dy\,dx = -\frac{5}{16} + \frac{1}{2}\ln(2)$$

2)

on va faire le changement en coordonnées cylindriques.

```
> x:=r*cos(theta);y:=r*sin(theta);
```

$$x := r\cos(\theta)$$

$$y := r \sin(\theta)$$

```
> assume(0<=r,r<=1,0<=theta,theta<=Pi/2);
> Int(Int(Int(r*(r**2+2*r*cos(theta)-1),z=0..r**2*cos(theta)*sin(t
  heta)),r=0..1),theta=0..Pi/2)=int(int(int(r*(r**2+2*r*cos(theta)
  -1),z=0..r**2*cos(theta)*sin(theta)),r=0..1),theta=0..Pi/2);
```

$$\int_0^{\frac{\pi}{2}} \int_0^{1} \int_0^{r\sim^2 \cos(\theta\sim)\sin(\theta\sim)} r\sim (r\sim^2 + 2\, r\sim \cos(\theta\sim) - 1)\, dz\, dr\sim\, d\theta\sim = \frac{11}{120}$$

```
> restart;# sol exo 7
```

On va faire un changement en coordonnées sphériques.

```
> x:=r*sin(phi)*cos(theta);y:=r*sin(phi)*sin(theta);z:=r*cos(phi);
```

$$x := r \sin(\phi) \cos(\theta)$$

$$y := r \sin(\phi) \sin(\theta)$$

$$z := r \cos(\phi)$$

```
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected

> assume(R>0,0<=r,r<=R,0<=phi,phi<=Pi/2,0<=theta,theta<=2*Pi);
> jacobian([x,y,z],[r,phi,theta]);
```

$$\begin{bmatrix} \sin(\phi\sim)\cos(\theta\sim) & r\sim \cos(\phi\sim)\cos(\theta\sim) & -r\sim \sin(\phi\sim)\sin(\theta\sim) \\ \sin(\phi\sim)\sin(\theta\sim) & r\sim \cos(\phi\sim)\sin(\theta\sim) & r\sim \sin(\phi\sim)\cos(\theta\sim) \\ \cos(\phi\sim) & -r\sim \sin(\phi\sim) & 0 \end{bmatrix}$$

```
> det(%);
```

$$\sin(\phi\sim)^3 \cos(\theta\sim)^2 r\sim^2 + \sin(\phi\sim)^3 \sin(\theta\sim)^2 r\sim^2 + r\sim^2 \cos(\phi\sim)^2 \cos(\theta\sim)^2 \sin(\phi\sim) + r\sim^2 \sin(\phi\sim) \sin(\theta\sim)^2 \cos(\phi\sim)^2$$

```
> simplify(%);# La valeur absolue est inutile, c'est déjà positif.
```

$$\sin(\phi\sim)\, r\sim^2$$

```
> i:=Int(Int(Int(r**2*sin(phi)*r*cos(phi)*cos(r**2*sin(phi)**2),r=
  0..R),theta=0..2*Pi),phi=0..Pi/2)=int(int(int(r**2*sin(phi)*r*co
  s(phi)*cos(r**2*sin(phi)**2),r=0..R),theta=0..2*Pi),phi=0..Pi/2)
  ;
```

$$i := \int_0^{\frac{\pi}{2}} \int_0^{2\pi} \int_0^{R\sim} r\sim^3 \sin(\phi\sim)\cos(\phi\sim)\cos(r\sim^2 \sin(\phi\sim)^2)\, dr\sim\, d\theta\sim\, d\phi\sim = -\frac{1}{2}\pi\cos(R\sim^2) + \frac{\pi}{2}$$

2) Indication : Faire un changement en coordonnées sphèriques pour (x/a,y/b,z/c).

```
> restart;# sol exo 8
```

On paramètre la courbe C par :

```
> assume(b>0,a>b);
> x:=theta->b*cos(theta);y:=theta->b+b*sin(theta);z:=theta->sqrt(2
  *y(theta)*(a-b));
```

$$x := \theta \to b\cos(\theta)$$

$$y := \theta \to b + b\sin(\theta)$$

$$z := \theta \to \sqrt{2\, y(\theta)\,(a-b)}$$

```
> int((y(theta)**2+z(theta)**2)*D(x)(theta)+(z(theta)**2+x(theta)*
  *2)*D(y)(theta)+(x(theta)**2+y(theta)**2)*D(z)(theta),theta=0..2
  *Pi);
```

$$-2\, b\sim^2 a\sim \pi$$

```
>
```